

The idea of vortex energy

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This work formulates and gives grounds for general principles and theorems that question the energy function doctrine and its quantum version as a genuine law of nature without borders of adequacy. The emphasis is on the domain where the energy of systems is conserved – I argue that only in its tiny part the energy is in the kinetic, potential and thermal forms describable by a generalized thermodynamic potential, whereas otherwise the conserved energy constitutes a whole linked to vortex forces, and can be a factor of things like persistent currents and dark matter.

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The motivation and scope

The physics of phenomena is chiefly perceived through the interactions given by the energy function in line with the principles of holonomic mechanics and equilibrium thermodynamics. This brilliantly unifying guideline created by Euler and Lagrange has found way in all pores of physics, and interpretations have spread out as if the guideline is a genuine universal law of nature. But with no outlined borders of adequacy, the law is a default belief, a source of circular theories and fallacies.

In this regard, worth recalling the forces called circulatory or vortex with all their cumulative impact beyond the energy function pattern that can be huge, as exposed since 19th century, e.g. [1,2] and the byword “dry water” stuck to viscosity-neglect hydrodynamic studies as inadequate, see Feynman lectures [3]. Also since 19th century, e.g. [4,5], the failure of the pattern was exposed in mechanics and other fields due to the reaction forces of ideal non-holonomy, performing no work on the system, as is the case of rigid bodies rolling without slipping on a surface. Recall also a general symmetry argument provoked by H -theorem of Boltzmann and showing the fundamental reversibility Loschmidt’s paradox [6] on the way to conform the real world with the energy function pattern.

The physics of nowadays for all that, now in line with the quantum mechanics claimed as more genuine than that of classical mechanics, further spreads out the conviction in the genuine energy law with no outlined borders of adequacy. And it has a commanding influence on both fundamental and applied research. This common trend has various sophisticated possibilities to fall into the same trap of circular theories, which in my judgment occurs here and there mainly due to playing with concepts of entropy and energy.

Indeed, “You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already

has a name. In the second place, and more important, no one really knows what entropy really is, so in a debate you will always have the advantage” as John von Neumann remarked on in another connection [7]. Curiously enough, it no less concerns what energy really is. This note will try to sketch my approach to it and the perspective on the energy perceptions I came to on this way; here it is through formulating and giving ground for relevant general principles and theorems. They rectify and develop the idea of energy duality claimed with insufficient argument in [8] in connection with the strong vortex effect of high frequency fields, and point clear to the conserved energy linked to vortex forcing which is complementary to all forms of energy function edifice.

The “energy cake” dilemma and equal footing theorem

The established consistent pattern of the world around is basically relaxation to roughly recurrent trends. It implies the ubiquity of irreversible forces as the generalized forces whose infinitesimal work depends on the path of system motion rather than just its instant state. An important mechanism is a manifold back-reaction of media.

May one then refer the irreversible forcing to the averaging of irrelevant variables of a conservative many-body system given by a microscopic Hamiltonian and random initial conditions? The answer to this widespread cue is no [8]. The cue misleads in the question of both statistical and dynamical (asymptotic over fast motion) averaging – there is no way to come to the irreversible behaviors from the formalism of energy functions unless resort to the inexact reasoning residing in the averaging methods and truncations irreducible to the separation by canonical transformations.

At the same time the perception of myriad of outer influences even treated as time-varying Hamiltonian interactions is inevitably via smoothing which barely complies with the exact separation given by canonical transformations, hence, contributes to the irreversible forcing along with arbitrariness in modeling the trends. This is like eat cake and have it: On the one hand, the irreversible forces,

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unlike reversible, cannot be derived from a Hamiltonian or effective potential. On the other, insofar as the true physics of phenomena is perceived through the interactions given by energy functions, so should be the physics of irreversible phenomena.

The “energy cake” dilemma formulated above is inherent to the perception and it imports fundamental inexactness in reasoning in terms of energy function. There is no other way to account for that but to integrate the energy formalism with a tentative (statistical) measure of energy blur/relaxation rates. This element pertains to both classical and quantum mechanics descriptions. The uncertainty principle of the latter is related to the postulated discreteness of energy transfer, has nothing to do with the dilemma, and the integration in point puts both descriptions on an equal footing.

Many phenomena in radiation and rays, superconductivity and other fields are commonly referred to as indescribable classically, which might be in one’s rights within some specific context; as for unconstrained assertions, it is to be questioned since contradicts the above theorem of equal footing. The same concerns the ideas of quantum computing claimed beyond classical physics, if their principles appear to be true.

Naturally, each of the two mechanics integrated with the element of diffusion/relaxation has a niche where it is handier depending on interest to discrete or continuous sides of phenomena. Even for such irreversible phenomena as highly deep cooling of matter by hf resonance fields both ways have led to its independent prediction, see [9], and we found the classical way direct, free of any linkage to the uncertainty principle defined by Planck constant. The point is not so much that this constant is the same for any nature of canonically conjugated variables, it is that the physics of phenomena can be perceived through any self-sufficient construction and that one can’t see through its wall unless allowing for a dual of the formalism with respect to wider frameworks.

The entrainment theorem

The inexactness of the trends prescribed by an energy function unfolds generally not only diffusion-like but also exponentially, able to radically change the system’s state, its stability and fluctuations. The vortex forces act so. The Gibbsian thermodynamics and the theory of generalized thermodynamic potential [10,2] commonly accepted in the study of phase transitions, transport through barriers, etc. abstract away from that. The generalized potential of a system relaxing in steady conditions to a density distribution ρ_{st} connects to it by

$$\rho_{st}(z) = N e^{-\Phi(z)}, \quad N^{-1} = \int e^{-\Phi} d\Gamma \quad (1)$$

where the integral is over the volume Γ of system phase space variables z and the reversible motion is on surfaces

$$\Phi(z) = const. \quad (2)$$

The properties of the system mainly depend then on the local properties of the minima of Φ . The analogous approach to systems under high frequency fields is in terms of the picture where the hf field looks fixed or its effect is time-averaged. In all this, Eq. (1) can be viewed as merely redefining the distribution ρ_{st} in terms of function Φ , which is suitable for the notion of the entropy of system states, whereas taking this function as the energy integral of reversible motion provides the physical basis of the theory, but implies rigid constraints.

Commonly, the constraints are reasoned as detailed balance (of transition probabilities between each pair of system states in equilibrium) within the framework of autonomous Fokker-Planck equations under natural boundary conditions by means of division of the variables and parameters into odd and even with respect to time reversal, with a reserve on factors like magnetic field. This logic, however, is model-bound and ill-suited unsteady conditions. Also it begs a question since detailed balance is in leading strings, for the reserve is not universal, e.g., breaks down in nuclear processes.

A different approach to outlining the overall domain of exactness was suggested in [8] and will be developed here. As in general, the basis is in keeping to invariance under transformations of variables. Obviously, $\Phi(z)$ to be the integral of reversible motion must be invariant under univalent transformations $z \rightarrow Z$, of Jacobian

$$|\det\{\partial Z_k(z, t)/\partial z_i\}| = 1 \quad (3)$$

where i, k run all components of z and Z , for then not only $\rho d\Gamma$ is invariant (being a number) but also $d\Gamma$ is.

The environment as a fluctuation/dissipation source for the system causes another invariance. Connecting Φ to the system’s energy function implies scaling this function in terms of environmental-noise energy level. The energy scale set so must vary proportionally with the energy function in arbitrary moving frames $Z = Z(z, t)$ to hold Φ invariant. Since the energy function changes in moving frames, this constraint can hold only for the systems *entrained* – carried along on the average at any instant for every system’s degree of freedom with the environment causing irreversible drift and diffusion.

Also account must be taken of that the limit of weak background noise poses as a structure peculiarity – transition to modeling of evolution without regard to diffusion. The entrainment constraint then keeps its sense as the weak irreversible-drift limit grasped via the scenarios of motion along the isolated paths in line with d’Alembert-Lagrange variational principle. Thereat, however, the principle still allows for the ideal non-holonomic constraints that violate the invariance of $\Phi(z)$. The invariance therefore necessitates the domain of entrainment free of that, termed ideal below.

We have reasoned about $\Phi(z)$ (1), but the reasoning holds for any one-to-one function of ρ_{st} . In unsteady conditions for the systems describable by a time-dependent density distribution $\rho(z, t)$, the adequacy of energy function formalism requires the entrainment ideal also. The

arguments used above for the systems of steady $\rho_{\text{st}}(z)$ become there applicable with univalent transformations of $\rho(z, t)$ into t -independent distribution functions.

The converse is also true: the behaviors governed by a dressed Hamiltonian $H(z, t)$ imply the entrainment ideal and the existence of a density distribution $\rho(z, t)$. As the velocity of underlying motion, $\dot{z} = \dot{z}(z, t)$, is constrained by $\dot{z} = [z, H]$ with $[\cdot, \cdot]$ a Poisson bracket, the divergence $\text{div} \dot{z} = \text{div}[z, H] = 0$ and $\text{div}(\dot{z}f) = -[H, f]$ for any smooth $f(z, t)$. It implies

$$\partial\rho/\partial t = [H, \rho] \quad (4)$$

which determines $\rho(z, t)$ from a given initial distribution and the natural boundary conditions preserving the normalization and continuity, for all other constraints are embodied in H . In no way the solution to (4) ceases to exist as unique, non-negative and not normalizable over the phase space of z where $H(z, t)$ governs the behaviors. The entrainment ideal there takes place since the solution turns to $\rho(H)$ in the interaction picture where H is t -independent. This completes the proof.

Thus, the necessary and sufficient conditions where the energy function doctrine is duly adequate to the evolution described by distribution functions come down to the entrainment ideal. This theorem lays down the overall domain of energy function adequacy sought for. It includes the systems isolated or in thermodynamic equilibrium, as well as entrained in steady or unsteady environments generally of non-uniform temperature or indescribable in temperature terms so long as the diffusion, irreversible drift and ideal nonholonomy can be neglected.

Remark 1. The trend of entrainment ideal can be deprived of evidential force already in the close vicinity of the ideal. In steady condition this can be not so much due to extremely long observation times as due to ideal nonholonomy, for the diffusion and irreversible drift are then enhanced hugely. To see it, suffice to bear in mind that the ideal nonholonomy not only reduces the number of degrees of freedom relative to the number of generalized coordinates, but also gives rise, see [5], to equilibrium states and also steady-motion states that are not isolated but form manifolds of one or more dimensions and asymmetry of secular-equation determinants.

Remark 2. The evolution of density distribution $\rho(z, t)$ of system states from a given initial $\rho(z, 0)$ gives by itself no insight into the matter of entrainment even in steady conditions, unlike, say, their multitude from various initial distributions. The trend of evolving then to one and the same shape of ρ means relaxation with the mean (“drift”) irreversible forces a factor. For the relaxation to steady motion, such forces are of vortex type in Γ as their forcing toward the steady motion and against it differ in sign. They effect both the steady and the transient shapes of ρ . Thereby the energy integral of the motion ceases to exist with time if set initially, being disrupted by the drift vortex forces along with diffusion. These forces act generally stronger than diffusion, multiplicatively, and cannot be compensated by conservative

forces. Unstable are then as minima of $\Phi(z)$ (1) as any points shifted from them, and the motion states can appear quite apart from the minima, in defiance of the theory of phase transitions based on generalized potentials.

The “energy - energy function” dualism and the conserved energy linked to vortex forcing

Let us refine on concepts. The issue of energy we are raising relates to the systems of finite degrees of freedom that interact with the environment whose influences of short correlation time are accounted for via the notion of entrainment introduced above. The systems are assumed describable by a smooth evolution of the density distribution $\rho(z, t)$ of phase space states z , a set of continuous variables $z = (x, p)$ with the generalized coordinates $x = (x_1, \dots, x_n)$ and conjugated moments $p = (p_1, \dots, p_n)$ of proper n taken in neglect of the constraints breaking the energy function formalism; z may include countable sets of normal mode amplitudes of waves in continuous media. The smoothness of ρ means

$$\partial\rho/\partial t = -\text{div}(v\rho) \quad (5)$$

with $v\rho$ the $2n$ -vector flux of phase fluid at z, t . Eq. (5) turns into the evolution equation of $\rho(z, t)$ under natural boundary conditions with v treated as operator on $\rho(z, t)$ that accounts for all constraints on the phase flows; in neglect of all nonlocal and retarded constraints, v is generally a t -dependent field divergent in z .

For the evolution of ρ modeled by an equation of form

$$\partial\rho/\partial t = [H, \rho] + I \quad (6)$$

where $H = H(z, t)$ is now, unlike in Eq. (4), an arbitrary smooth function taken for a Hamiltonian, and I embodies all other interactions, we have

$$I = -\text{div}[(v - \dot{z})\rho]$$

with $\dot{z} = [z, H]$ the local velocity of Hamiltonian phase flow and I a canonical invariant. The invariance of I holds as in as off the entrainment ideal. Indeed, a canonical (univalent) transformation $z \rightarrow Z$ implies not only the invariance of ρ and Poisson brackets but also the constraint $\partial Z(z, t)/\partial t = [Z, G]$ with G a function of z, t . Hence, on transforming Eq. (6) we get

$$(\partial\rho/\partial t)_Z = (\partial\rho/\partial t)_z - [G, \rho]$$

where the term $[G, \rho]$ is to be united with $[H, \rho]$ of (6), for just so a Hamiltonian is to change. Thus, Eq. (6) in the new variables differs by its r.h.s. changing so

$$[H, \rho] + I \rightarrow [H + G, \rho] + I \quad (7)$$

with I to be held invariant. This completes the proof.

It should be underlined that the invariance of I is not unconditional but under canonical transformations and

reflects the fact of proceeding in modeling from a Hamiltonian. Since it is governed by a Hamiltonian only in the entrainment ideal, only there I reduces to an invariant Poisson bracket; whereas the irreducibility of I this way beyond the ideal exposes I of Eq. (6) as the source of irreversible, hence, vortex forcing that breaks the invariance of I for any choice of $H(z, t)$.

Let us now turn to the concept of energy within the framework under study. At that, while the x, p of $\rho(z, t)$ is a set of phase space variables, the principle of virtual work on the system and the law of energy conservation, which are to be taken as prime as so the material world is perceived, are formulated in terms of isolated paths with x and p the functions of t . Naturally, we treat any conceivable isolated paths as abstraction of the kinetics of ρ , so the integrable correspondence between the two descriptions is to imply on the principles of continuity and causality. The n components ($v_{n+1}, v_{n+2}, \dots, v_{2n}$) of the *actual* phase flux at z, t act then as the generalized force conjugated to x and the scalar product

$$v_{n+1}\delta z_1 + v_{n+2}\delta z_2 + \dots v_{2n}\delta z_n \quad (8)$$

represents the virtual work on the system irrespective of whether this sum is reducible to the variation of a scalar function or not. Accordingly, for the generalized coordinates taken without restricting the generality in the geometric conditions not involving time explicitly, the density power on the phase fluid comes down to the scalar product

$$(v_{n+1}v_1 + \dots v_{2n}v_n)\rho. \quad (9)$$

In particular, the energy of the system is conserved as long as the integral of density power (9) over the whole phase volume holds zero,

$$\int (v_{n+1}v_1 + \dots v_{2n}v_n)\rho d\Gamma = 0. \quad (10)$$

This criterion bears by itself no relation to the entrainment ideal and shows up in both entrained and non-entrained systems and as under steady constraints (autonomous Eq. (5)) as unsteady.

Where the energy of system is conserved, there its energy measure exists in strict sense. So, the criterion (10)

outlines the existence domain of the energy measure. It includes the whole existence domain of the energy measure in the entrainment ideal, which is obviously where v is a t -independent divergent-free function of z , but can extend fairly far beyond it - however far in principle both in drift and diffusion terms of v and whether they are retarded and t -independent or not.

Consider in this light the conditions of energy conservation in the systems governed by autonomous Eq. (6). The branch I there acts on a par with $[H, \rho]$ in keeping the circulation and transformations of conserved energy, and this takes place as within as beyond the scope of entrainment ideal. This fact means that the energy circulating in conditions beyond the ideal is indescribable in terms of energy function. We called it vortex form of energy. The energy exchange between degrees of freedom is then not via detailed balance, involves vortex forcing. At that, the stationary conditions of ideal non-holonomy can be viewed as a particular case of such form of energy circulation.

Thus, unlike the conventional kinetic and potential energies and the thermal energy as the chaotic variant of kinetic energy within the generalized thermodynamic potential, the form of conserved energy circulating in the non-entrained systems is irreducible to a scalar function of system states. This form of energy is integral - relegation of its parts to conventional is not uniquely defined, which shows the vortex form of energy as complementary to the energy of conventional forms. This is what we call the energy - energy function dualism. It has nothing to do with the particle - wave dualism in quantum mechanics. The notion of energy quantum levels as the quanta of physical substance related to specific system states loses then its strict sense, and so does the transfer of energy via energy quanta.

The ability of vortex forces to radically, cumulatively change the system's state, stability and fluctuations, as is the case of systems under high frequency fields, and the ubiquity of vortex forcing also under the restrictions of system energy conservation conveys a suggestion that the vortex form of conserved energy can be a factor of persistent currents and also puzzles like dark matter.

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